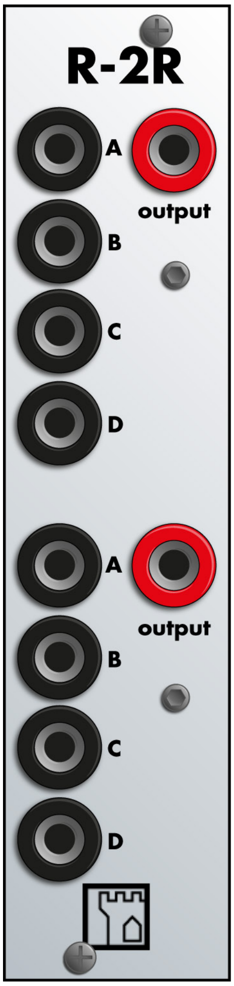
005 – R-2R

Two simple 4-bit analog to digital converters



Comments, suggestions, questions and corrections are welcomed & encouraged:
contact@castlerocktronics.com

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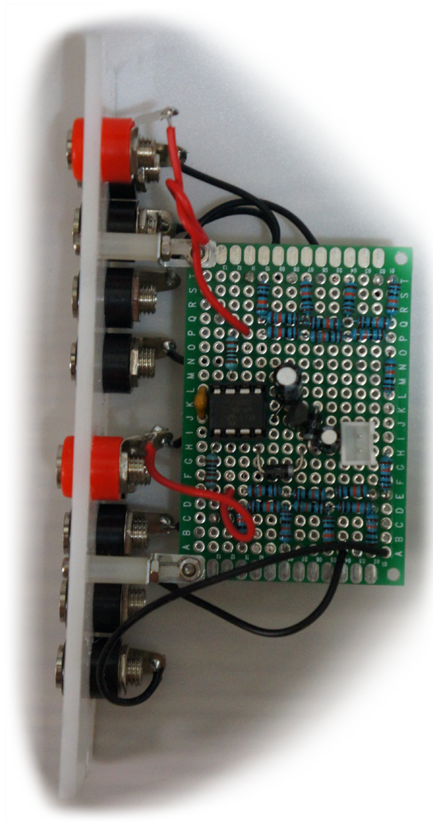
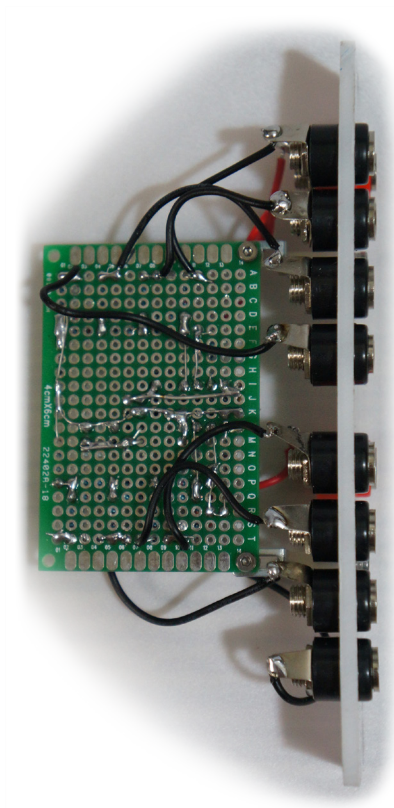
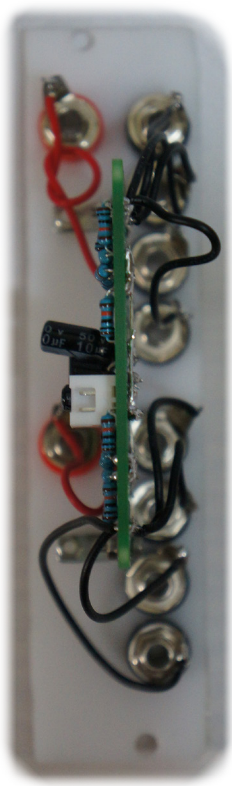
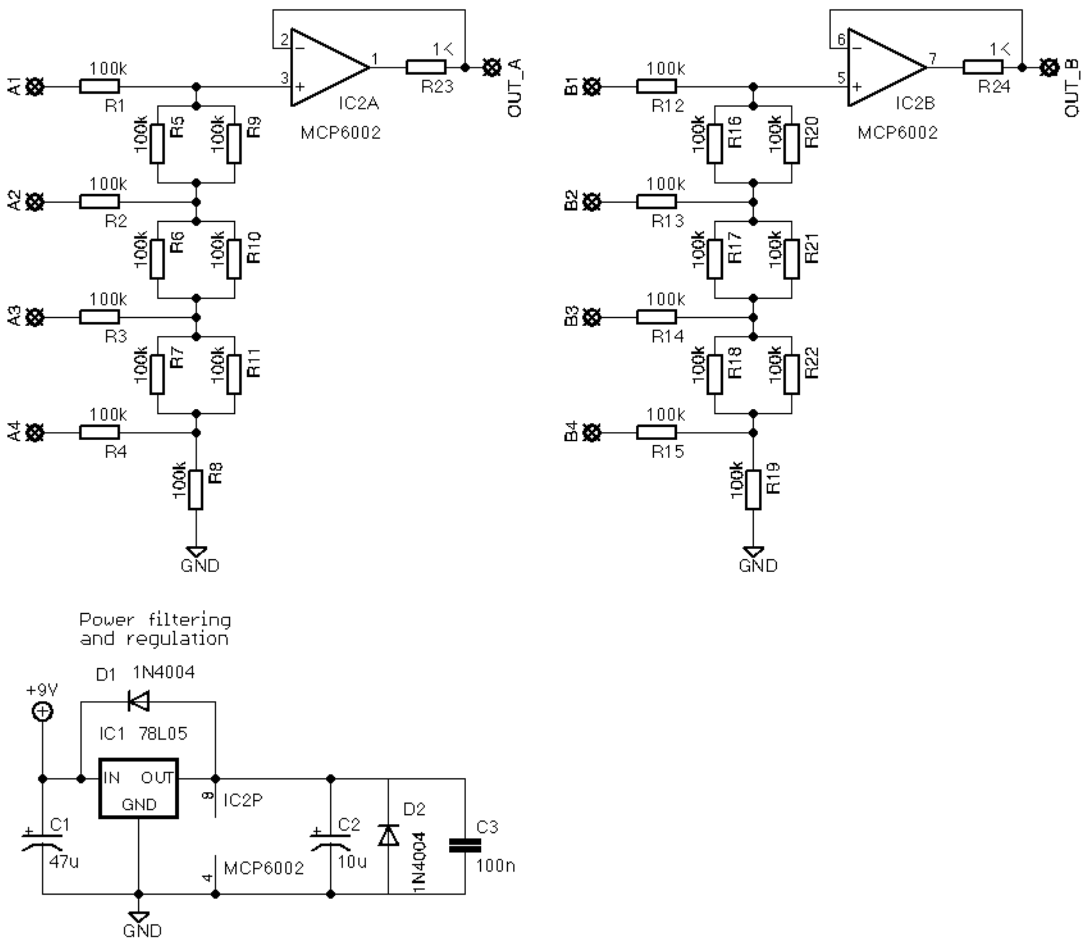
0.1 Photos







0.2 Schematic



0.3 Bill of Materials

ICs

78L05	*x1*
MCP6002	*x1*

Resistors

100kΩ	*x22*
1kΩ	*x2*

Capacitors

10μF	*x1*
47μF	*x1*
100nF	*x1*

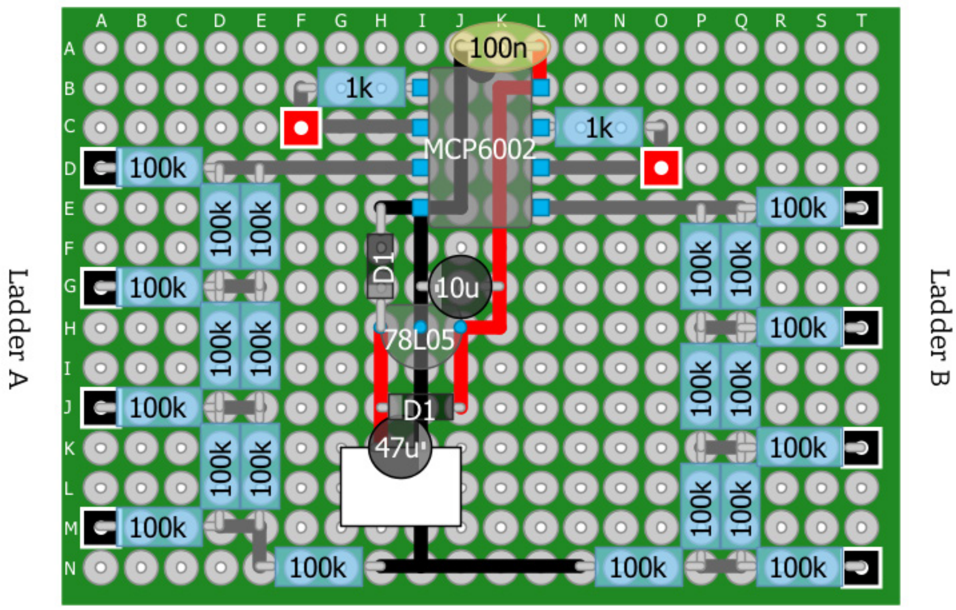
Diodes

1N4004	*x2*

Connectors

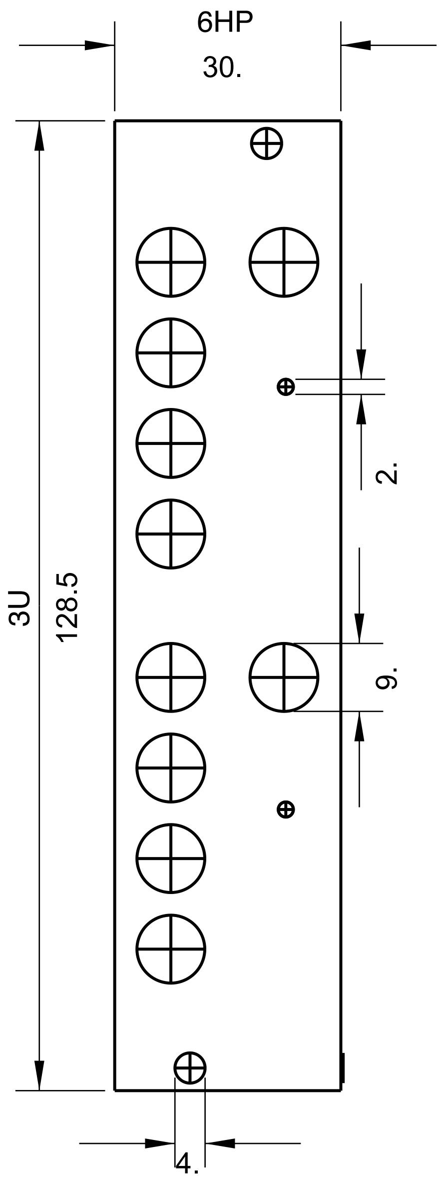
Banana Jacks	
Black	*x8*
Red	*x2*
XH–2.54	*x1*

0.4 Layout



Input
Output

0.5 Drill Template



All measurements in millimetres

1:1 scale

Print out to use

1. Explanation

1.1 The little DAC that could

A Digital To Analog Converter (DAC) is a circuit that takes one or more digital input sand then adds them together to make an analog voltage. There are many kinds of DAC and there are entire books dedicated to designing them. We're only interested in one though: the R-2R ladder.

An R2R ladder is probably the simplest and cheapest possible form of digital-to-analog converter. Made up of any number of parallel 1-bit inputs – meaning that every bit needs to be sent to a different input – each successive input contributes half as much as the input before it, with the first input itself only contributing half of what ever a logic-high is in the circuit – for us that is 5v, meaning that the first input will add 2.5v to the output of the R-2R ladder, the next 1.25v.

The name R-2R ladder comes from the fact that it only uses two values of resistor: R, which can be any value you want; and 2R, which is twice as big as R. This is what makes it so cheap and easy to build. Even better, putting two identical resistors in series is the same as a resistor of double the size while two identical resistors in parallel is the same as one of half the value. This means we can actually build it with only one resistor value, just as we have!

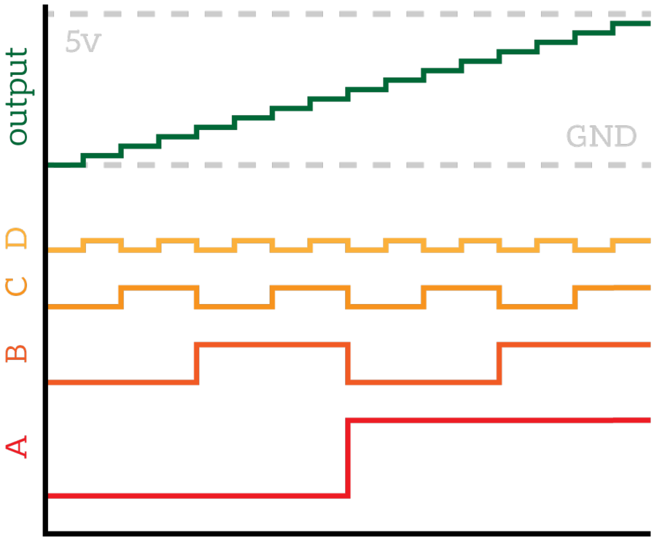
1.2 What does the output look like?

As we already mentioned, each input contributes half as much to the output as the one above it – no matter how many bits we add. To get a better idea of this take a look at *(fig 1.2a – next column)* to see how every combination outputs a different voltage.

As you can see, the R-2R ladder can let us use 4-digital inputs to get 2^4 (16!) different values.

You can also use analog outputs and use it as a weighted mixer for more fun!

1.3 The op-amp Buffer



D	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
C	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
B	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
A	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
output(V)	0	0.3125	0.625	0.9375	1.25	1.5625	1.875	2.1875	2.5	2.8125	3.125	3.4275	3.75	4.0625	4.375	4.6875

(fig 1.2a)

If you look at the schematic, you might also notice we have op-amps on the outputs. These act as buffers which let us connect the R-2R ladder's output to multiple other modules without much trouble at all. It also presents a high-impedance to the R-2R ladder so as to prevent us coming across any issues with low-impedance inputs loading down our ladder.

I chose to use the MCP6002 rail-ro-rail op-amp, a great choice for low-power single supply circuits like those in our modular. The actual output is not truly rail-to-rail but +- 25mv from its power supply rails. This is still pretty damn good though, and definitely good enough for our purposes.

It should also be noted that the MCP6002 is only rated up to 6v, so you don't want to be connecting it to the 9v power from our adaptor because you want a 0v – 9v output!

1.4 Word of warning

For reasons we will cover in the analysis section, every input needs to have something connected to it for the module to operate correctly. They also can not be diode-protected outputs! They must be able to both sink & source current.

2. Analysis

2.1 The skills we need to analyse the R-2R ladder

Since the R-2R ladder is a purely resistive circuit, calculating everything is actually relatively simple. The only problem is that we have to do the same kinds of calculations many times over. Ultimately though, everything is going to boil down to a healthy dose of Ohm's Law, Kirchhoff's Circuit Laws, voltage and current dividers, as well as superposition theorem – which isn't nearly as scary as it sounds.

2.1.2 Ohm's law

You've probably at least heard of Ohm's Law, if not already permanently etched it into the folds of your brain by using it over and over to convert currents to voltages and vice versa. For those of you who need a reminded, Ohm's Law describes the relationship between current and voltage as being defined by a property called "resistance". This useful measurement lets us figure out either current, voltage or resistance so long as we know two out of three.

$$V = IR \quad I = V/R \quad R = V/I$$

In our R-2R ladder, we already know the values of the resistors and the input voltage. From this we will be able to figure out the current flowing through each resistor and then from there we will use the current to calculate the voltage at other points.

2.1.3 Kirchhoff's Current Law

Probably less well known than Ohm's Law, but simpler and just as important is Kirchhoff's Current Law. Kirchhoff was a physicist who realised something that may seem a little obvious now – electric current can not simply appear or disappear in the middle of a circuit. This led Kirchhoff to make a bold, but true, statement:

The sum of currents flowing into any node of an electrical circuit is equal to the sum of the currents flowing out.

This little nugget of wisdom means that any time we are faced with a circuit that branches off in different directions, we can use this knowledge that the current flowing in and out of that point is equal to work on figuring out the current through each of the separate branches.

2.1.4 Kirchhoff's Voltage Law

Kirchhoff also had another very important law which is similarly simple and equally useful:

The directed sum of voltages in any closed loop will always equal zero

Special attention should be paid to the word "directed" here. This means that you have to take into account the DIRECTION of the voltage drop for it to count. When take your power supply voltage and subtract all the voltage drops across separate elements in the circuit you will always come out with 0, unless you have made a mistake. Similarly, when you go around a closed loop you have to go round in one direction and change the sign of your voltage drops from positive to negative when you start making your way from the lowest voltage point to the highest voltage point.

2.1.5 Series resistance and voltage dividers

Another thing that you have probably run into, but that we should do a little recap of, is what happens when you put resistors in series. Resistors in series behave exactly the same as a larger resistor equal to the sum of their individual resistances. All we have to do is add the resistances together!

$$R_{total} = R_1 + R_2 + \ldots + R_n$$

When we take this together with Ohm's Law and Kirchhoff's laws we can figure out exactly what the voltage between any two resistors, or even along multiple points in a string of resistors in series. Even better, we can set voltages in our circuits accurately by picking just the right resistors. Potentiometers have three terminals just for this purpose. They allow us to scale, attenuate and dial in voltages manually but acting as adjustable voltage dividers. It's this function of pots that allows us to use them as passive volume controls and passive attenuators, which both ultimately do the same thing.

For a simple two resistor voltage divider, the voltage at the junction can be calculated with this equation:

$$V_{out} = V_{in} \times \frac{R_{bottom}}{R_{top} + R_{bottom}}$$

This can be proven using Ohm's law. First we sum the resistors to figure out how much current flows through them together since I = V/R

$$I = \frac{V_{in}}{R_{top} + R_{bottom}}$$

Then we can find V_{out} by calculating how much of a voltage drop that amount of current causes through the bottom resistor using V = IR

$$V_{out} = I \times R_{bottom}$$

2.1.6 Resistors in parallel and current dividers.

Parallel is a little more confusing however. The RECIPROCAL of the total resistance is equal to the RECIPROCAL of each resistor added together. Reciprocal is basically a fancy word that means the number you would get if you divided 1 by the first number. So the reciprocal of 4 is ¼ (0.25) and the reciprocal of 10 is $^1/_{10}$ (0.1). So for every resistor, you take its value and then divide one by that number and write it down. When you have done this for every resistor, add all of those numbers together then divide 1 by that number:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

Another way to think of it if this is a little is to think about it in terms of "conductance". Conductance (indicated with a G) is measured in Siemens – don't laugh – and is not really used very often, but it is the reciprocal of resistance:

R = 1/G

G = 1/V

If you think about it this way, while you add the resistance of resistors in series, you add their conductance when they are in parallel. Resistors in series will conduct less, as less current flows for the same voltage drop. Resistors in parallel conduct more as each resistor will conduct the amount of current it would if it was the only resistor – so the total current flowing is more, making it the same as a lower resistance.

There are also two special situations with parallel resistors. We've already covered one of them: When two resistors of the same value are in parallel, the equivalent resistance is half of that value. However, when they are not the same, but there are still only two, then it is still a little easier to calculate. We even have a special symbol we can use to indicate the two resistors in parallel sum:

$$R1 \parallel R2 = \frac{R_1 R_2}{R_1 + R_2}$$

Using Kirchhoff's Laws, we can use resistors in parallel to build current dividers (fig 2.1.6a). Knowing that the current flowing out of a node must be equal to the current flowing in, we know that it must be split between the resistors in parallel. If we know the total current into the node, and the resistance of the resistor we want to know the current through, then we simply needed to use the following expression:

$$I_{R_1} = I_{total} \times \frac{R_2}{R_1 + R_2}$$

2.1.7 Superposition theorem

We're going to deal with this in more detail at the end of 2.2, but we'll give a brief overview right now. Superposition theorem is simply a way of dealing with circuits that have more than one voltage or current source. In our R-2R ladder, each of our inputs can be considered a separate voltage source and figuring out the current flowing through the resistors and the voltages at each node without superposition theorem is very difficult. Basically what we do is we pretend that all the voltage sources – except one – are actually connections to ground and then we figure out the currents and voltages this way, rinsing and repeating for each separate voltage source. For current sources, we pretend that they are actually an open circuit when we are not analysing them specifically. When we have done all the separate calculations, we then simply add them together, accounting for the direction they go, to figure out what happens when we have all of the sources involved in the circuit.

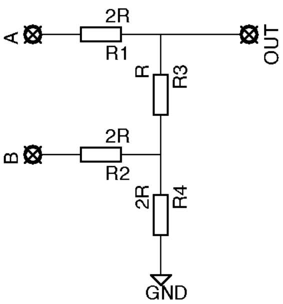
2.1.8 Making our calculations easier

Tragically, engineering professors do not hate this one weird trick of mine. This is no magic cure-all, but I find this trick really handy for doing analysis and trying to get your head around how things work. What we are going to do is replace all of our voltages and values with 1 because it is so much easier to do maths with 1V flowing through 1Ω resistors than it is to deal with 9V through 330kΩ resistors. Also, it means that all the values we will get in out calculations will be numbers we can use to multiply and scale the values we are actually using to figure out what results we should be expecting for the real thing. We'll do the actual scaling back up for our 5V inputs and 50kΩ/100kΩ resistors after building our four bit ladder.

2.2 Looking into the R-2R ladder from the output

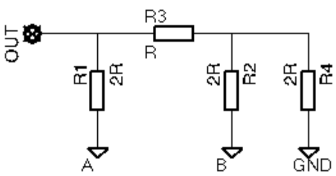
One thing that really deserves a mention before we start to look at the results that feeding bits into the ladder have is what resistance is seen from the output looking back in as its kind of ingenious and gives a bit of insight into why the R-2R ladder can be expanded to as many bits as you want.

2.2.1 2-bits



(fig 2.2.1a)

First lets look at what the input sees with all inputs grounded, but only two bits of input. First we need to draw up a simple 2-bit ladder *(fig 2.2.1a)*. From this point onward we are going to be using the letter A to identify our most significant bit (MSB) input – the one with the largest effect – and then B for the next most significant and C for the one below that and so on and so forth. "True" ground will still be denoted with GND.



(fig 2.2.1b)

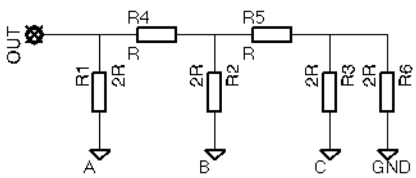
A nice way to simplify a resistive network to figure out what is in parallel and what is in series is to redraw it as a system of "branches" *(fig 2.2.1b)*, which is what we'll do every time from now on. If you take a look at the redrawn 2-bit ladder, with both inputs grounded and the output moved to the furthest left, you can see that the output sees a 2Ω resistor (R_1) in parallel with three other resistors made up of R_3 in series with R_2 and R_4 in parallel. Substituting everything in we can deduce that the total resistance must be:

$$R_{total} = 2\Omega \parallel (1\Omega + [2\Omega \parallel 2\Omega])$$

Remembering that a resistor in parallel with an equal resistor is equivalent to one resistor of half the size, we can do the maths easily.

$$R_{total} = 2\Omega \parallel (1\Omega + 1\Omega)$$
$$R_{total} = 2\Omega \parallel 2\Omega$$
$$R_{total} = 1\Omega$$

2.2.3 3-bits



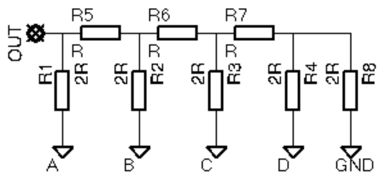
(fig 2.2.3a)

Doing the same as last time we can work out the resistance looking in from the output for a 3-bit ladder *(fig 2.2.3a)* with ease.

$$R_{total} = 2\Omega \parallel (1\Omega + [2\Omega \parallel \{1\Omega + (2\Omega \parallel 2\Omega)\}])$$
$$R_{total} = 2\Omega \parallel (1\Omega + [2\Omega \parallel \{1\Omega + 1\Omega\}])$$
$$R_{total} = 2\Omega \parallel (1\Omega + [2\Omega \parallel 2\Omega])$$
$$R_{total} = 2\Omega \parallel (1\Omega + [2\Omega \parallel 2\Omega])$$
$$R_{total} = 2\Omega \parallel (1\Omega + 1\Omega)$$
$$R_{total} = 2\Omega \parallel 2\Omega$$
$$R_{total} = 1\Omega$$

Are the results freaking you out yet?

2.2.4 4-bits and beyond



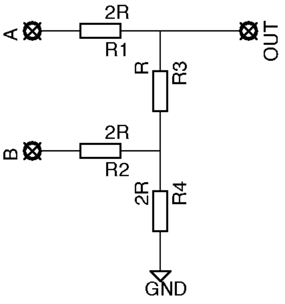
(fig 2.2.4a)

I'm sure you might have something of a hunch as to where this is going, but were going to make sure it really sinks in. Here is a 4-bit ladder redrawn, looking in from the output *(fig 2.2.4a)*

$$R_{total} = 2\Omega \parallel (1\Omega + [2\Omega \parallel \{1\Omega + (2\Omega \parallel [1\Omega + \{2\Omega \parallel 2\Omega\})\}])$$
$$R_{total} = 2\Omega \parallel (1\Omega + [2\Omega \parallel \{1\Omega + (2\Omega \parallel 2\Omega)\}])$$
$$R_{total} = 2\Omega \parallel (1\Omega + [2\Omega \parallel 2\Omega])$$
$$R_{total} = 2\Omega \parallel 2\Omega$$
$$R_{total} = 1\Omega$$

As you can see, no matter how many bits we add to the R-2R ladder, the output sees the exact same resistance driving it.

2.3 2-bit R-2R Ladder



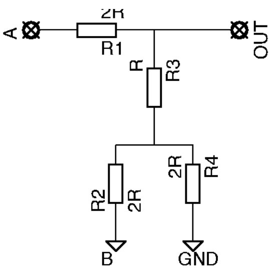
(fig 2.2.1a)

Before we tackle the whole ladder, we're going to wet our feet a little with a mini 2-bit ladder. *(fig 2.2a)* First we'll look at how much current flows through each resistor, and in what direction, for a high signal at both of the inputs separately. Then we'll use superposition theorem to find out what everything does when both inputs are high.

2.3.1 Input A high

Imagine that point A, our MSB input, is at 1V and all resistors in the network are only either 1 or 2. The first thing we are going need to do is figure out the total equivalent resistance looking in from that input. From that, we can calculate the total current flowing through the circuit, which we will then use in turn to figure out the current through each resistor and the voltage at the output point.

2.3.1.1 Total resistance and total current



(fig 2.3.1.1a)

Once more, the first thing we are going to do is redraw the circuit as a series of branches *(fig 2.3.1.1a)*. Input A is furthest on the left, and input B is grounded. From this we can see that the resistance seen at input A can be deduced as follows:

$$R_{total} = 2\Omega + 1\Omega + (2\Omega \parallel 2\Omega)$$
$$R_{total} = 4\Omega$$

From this, the total current that will flow if input A is brought up to 1V is easily calculated using Ohm's law:

$$I_{total} = \frac{V}{R} = \frac{1}{4}$$
$$I_{total} = 0.25A$$

2.3.1.2 Current through each resistor

The other advantage to the branch method comes when figuring out the current through each individual resistor. Thanks to Kirchhoff's current law we know that the current through any resistors in series must be the same, while resistors in parallel either have incoming current split between them or current coming out of them added together.

From this, we know that R_1 & R_3 must both have a whole 0.25A running through them, while R_2 & R_4 must have this same amount of current split between them equally – since their resistance is equal – making for 0.125A through branch of the current divider that they form.

2.3.1.3 Voltage at output

We actually have two paths to the answer here, we can treat the output as a point on a voltage divider, with R_1 forming the top and R_2 to R_4 forming the bottom; or we can calculate how much of a voltage drop must occur across R_2 – R_4 when 0.25A moves through them. Both of these need us to figure out the total equivalent of resistors R_2 – R_4, which is:

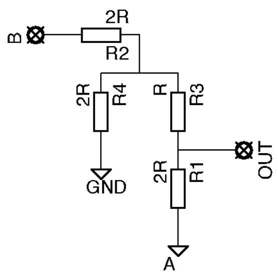
$$1\Omega + (2\Omega \parallel 2\Omega) = 2\Omega$$

According to the voltage divider method, since the input is 1V and the top and bottom have an equal resistance, the output is simply half of the input: 0.5V. This is also the same result we get from using Ohm's law to calculate the voltage drop across 2Ω when 0.25A passes through them:

$$V = IR = 0.25 \times 2 = 0.5V$$

2.3.2 Input B high

We're going to take the exact same approach to input B. Again, we are going to imagine point B is a 1V and point A is grounded.



(fig 2.3.2.1a)

2.3.2.1 Total resistance and total current

From the ladder we can use the same method as we did for input A to deduce that the resistance will be equivalent to:

$$R_{total} = 2\Omega + (2\Omega \parallel [1\Omega + 2\Omega])$$
$$R_{total} = 2\Omega + (2\Omega \parallel 3\Omega)$$
$$R_{total} = 2\Omega + 1.2\Omega$$
$$R_{total} = 3.2\Omega$$

With this result we simply use Ohm's law again to calculate the current:

$$I_{total} = \frac{V}{R} = \frac{1}{3.2} = 0.3125A$$

2.3.2.2 Current through each resistor

This time the answer is complicated slightly by the branches of the current divider that comes after R_2 is uneven. However, we can still easily figure out the current through R_2 itself, as it has to be the full 0.3125A. When it comes to the current divider it will be split between the two branches in proportion to their resistance. Lets first look at R_4 using our current divider equation from earlier. We will use R_x to refer to the other branch of the current divider made up of R_3 & R_1.

$$I_{R_4} = I_{total} \times \frac{R_x}{R_4 + R_x}$$
$$I_{R_4} = 0.3125 \times \frac{3}{2+3}$$
$$I_{R_4} = 0.3125 \times \frac{3}{5}$$
$$I_{R_4} = 0.3125 \times 0.6$$
$$I_{R_4} = 0.1875A$$

Now we have the current through R_4, all we need to do is subtract it from the total current for the current through R_1 & R_3, which will be the same as they are in series.

$$I_{R_{1\&3}} = I_{total} - I_{R_4}$$
$$I_{R_{1\&3}} = 0.3125 - 0.1875$$
$$I_{R_{1\&3}} = 0.125A$$

2.3.2.3 Voltage at output

The easiest way to calculate the voltage at the output is simply to calculate what voltage change occurs when 0.125A flows through R_1, which is 2Ω. Using Ohm's law, we can calculate that it will be 0.250V.

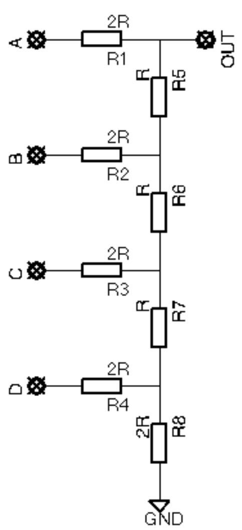
2.3.3 Both A & B high – Introducing superposition theorem

Now we want to know what the output is going to be when both of our inputs are high. This is where superposition theorem comes in. Thankfully, this has nothing to do with quantum physics and is far easier. All we need to do is add all the voltages and currents together from our two separate results, while considering the direction of current flow and whether the voltages are positive or negative – though this is something we don't have to worry about in our circuit.

For simplicity, we are going to put it all together in the form of a table of results *(fig 2.3.3a)*, with the direction of current flow indicated according to the original schematic, and not the redrawn ones we made to make figuring out the resistance easier:

A	B	current in mA, direction indicated according to *(fig 2.2.1a)*				V_{out}
		R_1	R_2	R_3	R_4	
0	1	→ 250	← 125	↓ 250	↓ 125	0.5
1	0	← 125	→ 312.5	↑ 125	↓ 187.5	0.25
1	1	→ 125	← 187.5	↓ 125	↓ 312.5	0.75

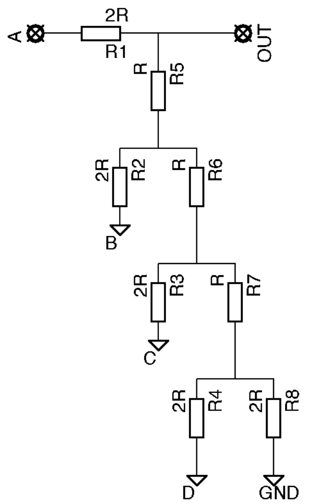
As you can see, after calculating the output voltage and current through the resistors for each the inputs separately, figuring out what the results will be when more than one is high is actually quite very simple thanks to superposition theorem.



(fig 2.4a)

2.4 4-bit ladder

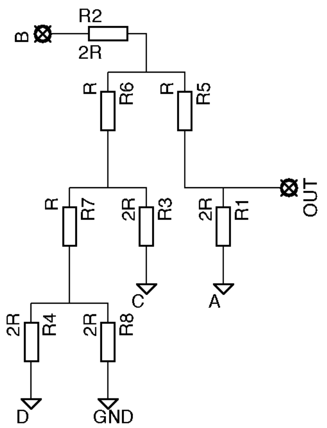
To save space and paper, but also to be lazy, we are not going to show the working for the calculations each individual output of the 4-bit ladder *(fig 2.4a)*. Instead, we are going to work through each input individually, showing the re-drawn branch version looking in from each of the four outputs, show the equivalent resistance looking in and the current through each resistor as well as the output voltage. You have all the tools necessary to do the calculations yourself if you want to, and doing so provides a great exercise for practicing the calculations we've already used.



2.4.1 Input A high

$$R_{total} = 2\Omega + 1\Omega + (2\Omega \parallel [1\Omega + \{2\Omega \parallel (1\Omega + [2\Omega \parallel 2\Omega])\}])$$
$$R_{total} = 4\Omega$$

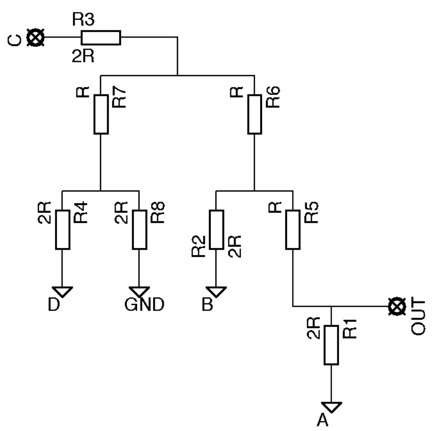
$$I_{total} = 250mA$$
$$V_{out} = 500mV$$



2.4.2 Input B high

$$R_{total} = 2\Omega + ([1\Omega + \{2\Omega \parallel 2\Omega\}] \parallel [1\Omega + \{2\Omega \parallel (1\Omega + [2\Omega \parallel 2\Omega])\}])$$
$$R_{total} = 3.2\Omega$$

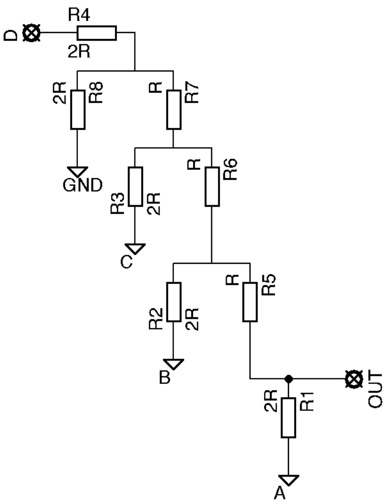
$$I_{total} = 312.5mA$$
$$V_{out} = 250mV$$



2.4.3 Input C high

$$R_{total} = 2\Omega + ([1\Omega + \{2\Omega + 2\Omega\}] \parallel [1\Omega + \{2\Omega \parallel (\Omega + 2\Omega)\}])$$
$$R_{total} = \sim 3.047\Omega$$

$$I_{total} = \sim 328.12mA$$
$$V_{out} = 125mV$$



2.4.4 Input D high

$$R_{total} = 2\Omega + (2\Omega \parallel [1\Omega + \{2\Omega \parallel (1\Omega + [2\Omega \parallel \{1\Omega + 2\Omega\}])\}])$$
$$R_{total} = \sim 3.012\Omega$$

$$I_{total} = \sim 332.03mA$$
$$V_{out} = 62.5mV$$

2.4.5 All possible combinations

A	B	C	D	current in mA, direction indicated according to *(fig 2.4a)*								V_{out}(mV)
				R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	
0	0	0	0	× 0	× 0	× 0	× 0	× 0	× 0	× 0	× 0	**0**
0	0	0	1	← 31.25	← 46.88	← 85.94	→ 332.03	↑ 31.25	↑ 78.12	↑ 164.06	↓ 167.97	**62.5**
0	0	1	0	← 63.5	← 93.75	→ 328.12	← 85.94	↑ 62.5	↑ 156.25	↓ 171.88	↓ 85.94	**125**
0	0	1	1	← 93.75	← 140.63	→ 242.18	→ 246.09	↑ 93.75	↑ 234.38	↓ 7.81	↓ 253.91	**187.5**
0	1	0	0	← 125	→ 312.5	← 93.75	← 46.88	↑ 125	↓ 187.5	↓ 93.75	↓ 46.88	**250**
0	1	0	1	← 156.25	→ 265.62	← 179.69	→ 285.16	↑ 156.25	↓ 109.38	↑ 70.31	↓ 214.84	**312.5**
0	1	1	0	← 187.5	→ 218.75	→ 234.38	← 132.81	↑ 187.5	↓ 31.25	↓ 265.62	↓ 132.81	**375**
0	1	1	1	← 218.75	→ 171.88	→ 148.44	→ 199.22	↑ 218.75	↑ 46.88	↓ 101.56	↓ 300.78	**437.5**
1	0	0	0	→ 250	← 125	← 62.5	← 31.25	↓ 250	↓ 125	↓ 62.5	↓ 31.25	**500**
1	0	0	1	→ 218.75	← 171.88	← 148.44	→ 300.78	↓ 218.75	↓ 46.88	↑ 101.56	↓ 199.22	**562.5**
1	0	1	0	→ 187.5	← 218.75	→ 265.62	← 117.19	↓ 187.5	↑ 31.25	↓ 234.38	↓ 117.19	**625**
1	0	1	1	→ 156.25	← 265.62	→ 179.69	→ 214.84	↓ 156.25	↑ 109.38	↓ 70.31	↓ 288.16	**687.5**
1	1	0	0	→ 125	→ 187.5	← 156.25	← 78.12	↓ 125	↓ 312.5	↓ 156.25	↓ 78.12	**750**
1	1	0	1	→ 93.75	→ 140.62	← 242.19	→ 253.91	↓ 93.75	↓ 234.38	↑ 7.81	↓ 246.09	**812.5**
1	1	1	0	→ 62.5	→ 93.75	→ 171.88	← 164.06	↓ 62.5	↓ 156.25	↓ 328.12	↓ 164.06	**875**
1	1	1	1	→ 31.25	→ 46.88	→ 85.94	→ 167.97	↓ 31.25	↓ 78.12	↓ 164.06	↓ 332.03	**937.5**

2.4.6 Extra credit challenge

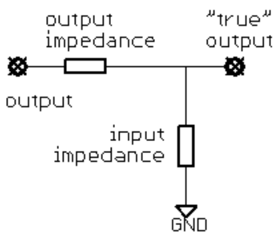
See if you can figure out the voltages at all the nodes along the ladder for each combination of inputs. A pretty brutal challenge, you may need a circuit simulator to check your results – I recommend the falstad circuit simulator for simple circuits where you also aren't interested in the frequency or phase response. It provides a nice "real-time" visualisation of current flowing through the circuit among other nice tools that can really help give you a better feel of what goes on inside a circuit.

2.5 Scaling our results back up

Now all that is left for us to do is scale our results back up to get the voltages and currents in the circuit we have actually build. For voltages, this is simple. All we need to do is multiply the output voltages from our table – remembering that they are in mV! – by the voltage that represents a logic high in our modular.

Calculating the currents is only slightly more difficult. First we multiply 5V by the value used for R in our ladder. Remember that R is 50kΩ in our ladder, NOT 100kΩ! This is because we use 100k for 2R and then put two in parallel to get 50kΩ. Using Ohm's law, we get that 5V across 50kΩ will cause 100μA of current to flow. Then all you need to do is multiply the values on the table by 100μA, again remembering that the numbers on the table are in mA and not A, so divide the numbers by 1000 to get them in A.

2.6 The effect of loading on the R-2R ladder



(fig 2.6a)

One last thing we ought to take a look at, though not in as much detail, is why we have output buffers. Obviously, we are going to want to connect our ladder to some sort of input so we can actually get use out of it. However, all inputs have something called "input impedance". In fact, outputs also have an output impedance. The two of these are important concepts to have at least some grasp of, they are terms we use to refer to the resistance seen by other things connected to inputs or outputs. We can think of an output's impedance forming a voltage divider together with the input impedance *(fig 2.6a)*.

From this you can see that a if we have a very low output impedance and a very high input impedance then the voltage at point x will be closer to the output that we want. However, if the output impedance is high and the input impedance low then the voltage at x is going to be much, much lower than what we wanted it to be.

So how high and how low are good enough? As a rough yardstick measurement, something like 1k output impedance and 100kΩ to 1MΩ input impedance will suffice. A output with an impedance of 1kΩ connected to an input with an impedance of 100kΩ will only loose about 1%, which is not bad at all.

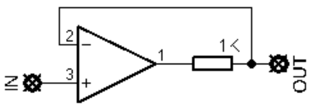
However, this wouldn't be good enough for our R-2R ladder as it is made up of 100k resistors itself. Remember that the resistance seen looking in from the output will always be equivalent to R, which for us is 50kΩ. This means that an input impedance of 100kΩ would be equivalent of a 100kΩ resistor in parallel with 50kΩ. Our actual output voltage would be around 1/3rd of what it ought to be!

Even worse is that the R-2R ladder is going to be very sensitive to the input impedance of whatever it is connected to – the voltage output will vary a great deal across different impedances. This is where our op-amp buffers come in.

2.7 The op-amp buffers

Buffers serve to let us convert a higher-impedance output into a lower-impedance one by first providing a high-impedance input to the original circuit to prevent loading – and then making a lower impedance copy of the signal at its output. Op-amps make fantastic buffers, they often have huge input impedances – the MCP6002 has in input impedance of 10TΩ (that's teraohms!) – while also having very low output impedances. Putting an op-amp buffer between the R-2R ladder and the actual output lets us ignore any problems we would have with loading all together, and makes the output consistent for a wide range of input impedances that we might connect it to.

If you remember building our 4-channel mixer



(fig 2.7a)

– *article 002* – you might also remember that op-amps work as hard as they can to make the voltage at the inverting input the same as the voltage at the non-inverting input. This makes creating a buffer out of an op-amp really simple *(fig 2.7a)*. All we have to do is connect the op-amp's output to the inverting input and feed the R-2R ladder's output to the non-inverting input on the op-amp.

However, we also want to add a current limiting resistor to protect against short circuits or anything else going wrong. I chose 1kΩ as it allows for a really low output impedance, which is great for stacking out banana cables and driving more than one input with the same R-2R ladder. 5V across 1kΩ is about 5mA, which is paltry compared to what the MCP6002 is rated for, so it's well within the safe range. You might also notice that we actually take the feedback from AFTER the resistor. This is so that the op-amp is going to make the voltage at the real output – which is after the resistor – equal to the voltage at it's non-inverting input. This is a really easy form of compensation that we can add to further ensure that our op-amp behaves as well as possible when connected to more than one input, as it will make up for any loading issues caused by connecting to many inputs in parallel.

3. Modifications

3.1 More bits

If you read through the analysis, you'll see that adding more bits is just as easy adding more rungs to the R-2R ladder. Add as many as you want! Each new input will contribute half as much voltage as the one above it.

However, one thing to bear in mind is that the more bits you add, the less accurate the ladder will be. This is due to the fact that even 1% resistors still have 1% of error and the R-2R ladder is very dependant on the accuracy of the resistors used to make it. For our synth though, it probably doesn't matter very much at all.